Simplex Method – Lecture Summary and Implementation (March 08)

# 1. Introduction to Simplex Method

• The Simplex Method is a powerful iterative algorithm used to solve Linear Programming Problems (LPPs) efficiently. It finds the optimal solution by moving along the edges of the feasible region in a stepwise manner.  
  
• Iterative process in Simplex method:  
 1. Identify the entering variable (most negative coefficient in the objective row).  
 2. Identify the leaving variable (smallest positive ratio in RHS).  
 3. Perform pivoting (row operations) to update the Simplex Table.  
 4. Repeat until no more negative coefficients exist in the objective row.

# 2. Learning Objectives

By the end of this session, you will be able to:  
• Define steps involved in Simplex Method  
• Solve different steps with an example  
• Python implementation of Simplex Method

# 3. Step-by-Step Simplex Method

Step 1: Convert Constraints to Equations

Objective Function:  
Maximize Z = 3x + 5y  
  
Subject to:  
 2x + y ≤ 8  
 x + 2y ≤ 6  
 x, y ≥ 0  
  
Introducing slack variables s1 and s2:  
 2x + y + s1 = 8  
 x + 2y + s2 = 6  
  
Final objective function:  
 Z = 3x + 5y + 0s1 + 0s2

Step 2: Set up Initial Simplex Tableau

Basis | x | y | s1 | s2 | RHS  
-----------------------------  
s1 | 2 | 1 | 1 | 0 | 8  
s2 | 1 | 2 | 0 | 1 | 6  
Z-row |-3 |-5 | 0 | 0 | 0

Step 3 & 4: Identify Entering and Leaving Variables

• Most negative coefficient in Z row is -5 → Entering variable: y  
• Perform ratio test:  
 S1: 8/1 = 8  
 S2: 6/2 = 3 → Leaving variable: s2 (smallest ratio)

Step 5: Pivoting

• Pivot element is 2 (intersection of entering variable y and leaving variable s2).  
• Row operations:  
 New Row 2 = Row 2 / 2 = (1/2, 1, 0, 1/2, 3)  
 New Row 1 = Row 1 - (1 × New Row 2)  
 = (2, 1, 1, 0, 8) - (1/2, 1, 0, 1/2, 3)  
 = (3/2, 0, 1, -1/2, 5)  
 New Z-row = Z + 5 × (New Row 2)

Step 6: Repeat Until Optimality

• Continue pivoting until all values in the Z row are non-negative.  
• Final solution:  
 x = 2, y = 3  
 Z = 21

# 4. Python Implementation (Scipy)

from scipy.optimize import linprog  
  
c = [-3, -5]  
A = [  
 [2, 1],  
 [1, 2]  
]  
b = [8, 6]  
x\_bounds = (0, None)  
y\_bounds = (0, None)  
result = linprog(c, A\_ub=A, b\_ub=b, bounds=[x\_bounds, y\_bounds], method="simplex")  
  
print("Optimal x:", result.x[0])  
print("Optimal y:", result.x[1])  
print("Optimal Z:", -result.fun)

# 5. Academic Poll Q&A

Q1: In the Simplex Method, how do we identify the entering variable?  
A) The variable with the most positive coefficient in the objective function row  
B) The variable with the most negative coefficient in the objective function row ✅  
C) The variable with the smallest absolute value in the constraint matrix  
D) The variable with the highest coefficient in any constraint

Q2: What is the purpose of the minimum ratio test in the Simplex Method?  
A) To determine which variable should enter the basis  
B) To decide the optimal value of the objective function  
C) To identify which variable should leave the basis ✅  
D) To check if the solution is feasible

Q3: Which condition indicates that the optimal solution has been reached in the Simplex Method?  
A) All elements in the last row (objective function row) are non-negative ✅  
B) The tableau contains a row with all zero values  
C) The pivot column has all negative values  
D) The right-hand side (RHS) contains only negative values

Q4: What happens if there is no feasible solution in the Simplex Method?  
A) The algorithm continues indefinitely  
B) The objective function value remains constant  
C) The tableau contains a row with a negative RHS and all non-positive coefficients ✅  
D) The method selects a random solution

# 6. Session Summary

• Different steps involved in solving LPP using Simplex Method  
• Implementing these steps in Python